MAXWELL ON THE METHOD OF PHYSICAL ANALOGY *

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IN addition to his scientific work, James Clerk Maxwell gave some attention to what is now known as the philosophy of science. Much of what he said has been said by others, but he did say it early and rather well. Maxwell was interested in both broad philosophical issues and the special problems which arose out of his own contribution to physics. These special problems included physical analogy, dynamical explanation, and action at a distance. The present report is confined to Maxwell's views on the method of physical analogy.

There are fashions in scientific explanation. Nineteenth-century Cambridge physics had its own distinctive cut, but Maxwell's observations on the methods he employed and the results he obtained are still instructive today. Maxwell did not write systematically on physical analogy. His views have to be gathered from the many asides that are scattered through his scientific memoirs and treatises, and from the more popular papers he prepared. These papers include lectures, addresses to learned societies, biographical sketches, articles for the Ninth Edition of the *Encyclopædia Britannica,* and book reviews for *Nature.*[1]

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[1] The scientific memoirs together with the papers of general interest are collected in the *The Scientific Papers of James Clerk Maxwell,* ed. by W. D. Niven, Cambridge, 1890. The treatises comprise *A Treatise on Electricity and Magnetism,* 3rd edn., Oxford, 1892 ; the *Elementary Treatise in Electricity,* ed. by William Garnett, Oxford, 1881 ; *Theory of Heat,* 8th edn., London, 1885 ; and, written for a wider audience, *Matter and Motion,* New York. Maxwell edited *The Electrical Researches of the Honourable Henry Cavendish, F.R.S.,* Cambridge, 1879. Some of Maxwell's letters and philosophical essays not written for publication are printed in Lewis Campbell and William Garnett, *The Life of James Clerk Maxwell,* London, 1884, ; and in *Origins of Clerk Maxwell's Electric Ideas,* ed. by Sir Joseph Larmor, Cambridge, 1936.

References to the *Papers* will be by volume and page ; references to *Electricity and Magnetism* and the *Elementary Treatise* will be by article ; references to the remaining works will be by page. The *Papers* and *Electricity and Magnetism* as well as *Matter and Motion* have recently been reprinted by Dover.

MAXWELL ON METHOD OF PHYSICAL ANALOGY

I

In an address delivered in 1870 to the British Association, Maxwell characterised physical analogy as a similarity in mathematical form between two branches of science which differed in the physical nature of the phenomena they described. He also referred to physical analogy as scientific illustration and as scientific metaphor.

> Now a truly scientific illustration is a method to enable the mind to grasp some conception or law in one branch of science, by placing before it a conception or a law in a different branch of science, and directing the mind to lay hold of that mathematical form which is common to the corresponding ideas in the two sciences, leaving out of account for the present the difference between the physical nature of the real phenomena.[1]

In the course of his writings Maxwell made three points that served to clarify the nature or character of physical analogy. One, he found physical analogy in scientific explanation to be the mean between pure mathematical formulae, on the one hand, and physical hypotheses, on the other. Two, he distinguished two senses in which analogies are incomplete, that is, two senses in which the similarity between the branches of science compared is only partial. Three, he discussed the possibility of a new language suitable to the exact, yet economical, expression of different physical analogies. Maxwell's comments were not formal. He both made these points and drove them home by means of well chosen illustrations.

The account of physical analogy as a mean between the excesses of mathematical description and physical hypothesis appeared in Maxwell's very first memoir on electricity and magnetism, his paper ' On Faraday's Lines of Force ', published in 1855. The opening paragraph complained that the mathematical study of electricity had become so complicated that its mere retention in the memory hindered the further progress of that discipline. Simplification was necessary.

> The results of this simplification may take the form of a purely mathematical formula or of a physical hypothesis. In the first case we entirely lose sight of the phenomena to be explained; and though we may trace out the consequence of given laws, we can never obtain more extended views of the connections of the subject. If, on the other hand, we adopt a physical hypothesis, we see the phenomena only through a medium, and are liable to the

[1] ' Address to the Mathematical and Physical Sections of the British Association,' *British Association Report*, 40; reprinted in *Papers*, 2, 219

> blindness of facts and rashness in assumption which a partial explanation encourages. We must therefore discover some method of investigation which allows the mind at every step to lay hold of a clear physical conception, without being committed to any theory founded on the physical science from which the conception is borrowed, so that it is neither drawn aside from the subject in pursuit of analytical subtleties, nor carried beyond the truth by a favourite hypothesis.[1]

The memoir used the analogy between light and wave motion in an elastic solid to illustrate this point. To strip the theory of light of its dressing, to reduce it to ' theory of " transverse vibration " ', is to offer a mathematical description. To ask what property of light it is that vibrates, or to seek a medium in which this vibration occurs, is to resort to physical hypothesis. But to exploit this comparison to the fullest and yet remember that it is but a similarity in mathematical form between two sciences, is to proceed by the method of physical analogy.[2]

The memoir ' On Faraday's Lines of Force ' applied this golden mean to the problems of electricity and magnetism. It translated some of Faraday's ideas into mathematical language, but it did not yet contain the general equations of the electromagnetic field. This discovery awaited Maxwell's second memoir, ' On Physical Lines of Force ', published in 1861-62. The translation of Faraday's ideas was accomplished, in part, by the demonstration of the existence of an analogy between what are now called stationary or steady fields and the steady motion of a frictionless, incompressible fluid. The fluid was imagined to move through a sponge-like substance which opposed its motion with a resistance proportional to the velocity of the fluid. Here in outline is this analogy for the electrostatic field.

The positively and negatively charged bodies are analogous respectively to the sources at which the ideal fluid is created and the sinks at which it empties. The lines of electric force are analogous to what Maxwell called ' tubes of fluid motion '. The lines of force indicate, by their direction and divergence (or relative compactness), the direction and intensity at any point of the electric force ; the tubes, running from the sources to the sinks, indicate, by their direction and cross sections, the direction and speed at any point of the fluid. The pressure exerted by the fluid in overcoming the resistance of the sponge-like substance is analogous to the electrostatic potential, while

[1] ' On Faraday's Lines of Force ', *Transactions of the Cambridge Philosophical Society*, 10, Part 1 ; reprinted in *Papers*, 1, 155-156

[2] Ibid., *Papers*, 1, 156

the substance itself is analogous to the dielectric in which the charged bodies are embedded.[1]

The analogy is of historical importance because it clearly indicated that electrostatics could be investigated mathematically, not only in terms of Coulomb's inverse square law, but also in terms of Faraday's lines of force. The lines of electric force and tubes of fluid motion both satisfy the same partial differential equations. This analogy, by the way, was not the first attempt to compare Faraday's lines of force to more familiar notions. Kelvin, in 1842, when seventeen years old, in his paper 'On the Uniform Motion of Heat in Homogeneous Solid Bodies,' established an analogy between electrostatics and the flow of heat in an unevenly heated solid.[2]

The analogy between electrostatics and fluid motion was an application of Maxwell's golden mean. The fluid was not offered as a physical hypothesis nor the theory developed in purely mathematical terms. 'On Faraday's Lines of Force' emphasised this fact.

> The substance here treated . . . is not even a hypothetical fluid which is introduced to explain actual phenomena. It is merely a collection of imaginary properties which may be employed for establishing certain theorems in pure mathematics in a way more intelligible to many minds and more applicable to physical problems than that in which algebraic symbols alone are used.[3]

And in another passage :

> In this outline of Faraday's electrical theories, as they appear from a mathematical point of view, I can do no more than simply state the mathematical methods by which I believe that electrical phenomena can be best comprehended and reduced to calculation,

[1] 'On Faraday's Lines of Force', *Transactions of the Cambridge Philosophical Society*, 10, Part 1 ; reprinted in *Papers*, 1, 160-175

[2] Lord Kelvin, 'On the Uniform Motion of Heat in Homogeneous Solid Bodies', *Cambridge Mathematical Journal*, 1842, 3, reprinted in *Reprint of Papers on Electrostatics and Magnetism*, London, 1872, pp. 1-14. Maxwell did refer to Kelvin's work. Kelvin, in turn, in a note added to the memoir in 1854, stated that he was anticipated in some results by Chasles who was anticipated by Green. The analogy to heat, however, Kelvin claimed, was not previously established. In another note, added the same year, Kelvin drew attention to Faraday's use of phrases like 'conducting power' in his discussion of lines of force. E. T. Whittaker in his *A History of the Theories of Aether and Electricity*, London, 1910, p. 269, remarks, however, that already in 1827 Ohm had found an analogy between the flow of heat and an electric current in a solid conductor, which is close to Kelvin's analogy.

[3] 'On Faraday's Lines of Force', *Papers*, 1, 160

> and my aim has been to present the mathematical ideas to the mind in an embodied form, as systems of lines or surfaces, and not as mere symbols, which neither convey the same ideas, nor readily adapt themselves to the phenomena to be explained.[1]

Maxwell's researches in the kinetic theory of gases may be understood as the development of a physical analogy into a physical hypothesis. The system of particles came to 'constitute', as Maxwell said, the gas in question. If the analogy of the ideal fluid were likewise changed into a physical hypothesis, it would be something like the effluvia theories of electricity offered by Gilbert in the seventeenth century and Desagulier in the eighteenth century. These thinkers pictured electrostatic attraction and repulsion as due to an imponderable medium streaming out of the charged bodies to act by contact upon other charged bodies.[2]

The second point which served to clarify the nature of physical analogy concerned the two senses in which analogies are incomplete. This material was best presented in Maxwell's very last work, the *Elementary Treatise on Electricity*, published posthumously in 1881. It was there illustrated in terms of Kelvin's analogy between electrostatics and heat conduction, but here it will be simpler to translate the illustration to the analogy between electrostatics and fluid motion. First, analogies were found incomplete in the sense that the one science has only a special group of instances which are analogous to the other science.[3] As already described, the analogy between fluid motion and electrostatics holds only for the special group of instances in which the motion of the fluid is steady and in which the fluid is frictionless and incompressible. There are additional limitations. The memoir 'On Faraday's Lines of Force' also pointed out that the fluid must fill completely the space under consideration. If there are islands, then the velocity of the fluid at a point will not vary inversely as the square of the distance from that point to a source.[4] Further, a continuous supply of energy from outside the system is required to

[1] 'On Faraday's Lines of Force' *Papers*, I, 187. See also Maxwell's letter to Kelvin, 13 September 1855, written during the preparation of the memoir: 'Now I have been planning and partly executing a system of propositions about lines of force &c which may be *afterwards* applied to Electricity, Heat or Magnetism or Galvanism, but which is in itself a collection of purely geometrical conceptions of lines, surfaces &c.' Here Maxwell referred to physical analogy as 'allegorical representation'. *Origins of Clerk Maxwell's Electric Ideas*, p. 17.

[2] For a description of these theories see E. T. Whittaker, *History*, pp. 29-31, 37-39.

[3] *Elementary Treatise*, § 65

[4] 'On Faraday's Lines of Force', *Papers*, I, 159

generate the fluid and drive it through the sponge to the sinks, while, as stated in the *Elementary Treatise on Electricity*, a system of charged bodies placed in a good insulator will remain electrified indefinitely.[1]

Second, the analogies were found incomplete in the sense that certain quantities describing one set of phenomena may represent, to use Maxwell's terminology, ' physical states ', while the analogous quantities describing the other set of phenomena are ' mere scientific concepts '.[2] If a quantity is connected to other effects which are independently defined, then it is a physical state ; if not, then it is a mere scientific concept. The velocity of the fluid at any point in the space surrounding the sources and sinks represents a physical state. A change in this velocity means a change in the distribution of sources and sinks. But it also means a change in other properties of the fluid which are connected to the velocity in determinate ways, for example, the temperature, the density, and the viscosity of the fluid. The electric force at a point in the space surrounding the charged bodies is a mere scientific concept. A change in the electric force means only a change in the distribution of the charged bodies themselves. The concept of a force at a point in space is simply a device which the mathematician employs to compute forces in certain complicated problems which arise in the study of the interaction of charged bodies.[3]

To put the distinction between electric force and velocity in a slightly different manner : the electric force at a point is defined as the force which would act upon a unit charge if placed there. No independent definition exists. The only way to determine this force is by an actual test charge or by a calculation from the distribution of charge producing the force. The velocity of the fluid at a point might be defined as the force which would act upon a small cork tied to a spring if placed there. Here independent definitions do exist. The velocity could be determined by an actual test with such a cork or by a calculation from the distribution of sources and sinks producing the velocity. But this velocity could also be calculated

[1] *Elementary Treatise*, § 65 [2] Ibid., § 66

[3] Maxwell made this distinction in terms of Kelvin's analogy. He compared temperature as a concept representing a physical state to electrostatic potential as a mere scientific concept. Elsewhere concerning the expressions in the electrostatic field for energy, tension, and pressure, Maxwell wrote : ' If the action of the system E_2 on E_1 does in reality take place by direct action at a distance, without the intervention of any medium, we must consider . . . (these) quantities as mere abbreviated forms for certain symbolical expressions, and as having no physical significance '. (*Electricity and Magnetism*), § 105

from other properties of fluid. For example, if the law connecting the temperature of fluid to its velocity were known, then the velocity of the fluid at a point could be calculated from its temperature at that point.

Incompleteness of analogy in the second sense is not the same as the distinction between physical analogy and physical hypothesis. It is logically possible to claim that a given quantity represents a physical state without making the further claim that some hypothetical substance constitutes the quantity in question. Fourier's *Théorie analytique de la chaleur,* for example, treats heat as an entity that travels, but nowhere names the vehicle. There is no hypothesis either to the effect that heat is the flow of the hypothetical substance called caloric or to the effect that heat is a mode of motion of hypothetical particles called molecules.

The third point which served to clarify the nature of physical analogy concerned the development of a new language suitable for the exact yet economical statement of different physical analogies. ' On Faraday's Lines of Force ' was written under the handicap of a language too vague for its precise ideas. The fluid picture in electrostatics was suggestive, but, since the analogy is incomplete, it was also misleading. Here for students of semantics is a good illustration of the ' tyranny of words ', a phrase Maxwell himself employed upon occasion.[1] A new language was needed to express the similarity between them. Vector analysis, and in particular, the analysis of vector fields, filled this need.[2] The development of vectorial methods first paralleled, then became part of, the development of electromagnetic theory. Hamilton, it will be remembered, published his *Lectures on Quaternions* in 1853, and P. G. Tait developed the various uses of the differential operator in the 1860s. Viewed in the light of Maxwell's later work, Faraday's employment of lines of force appear as a method of describing vector fields without the benefit of mathematical notation, and Maxwell's use of tubes of fluid motion appear as something halfway between Faraday's methods and vectorial methods. The analogy of the first memoir is easily expressed today. In electrostatics, the curl of the electric force equals zero ; in the motion of Maxwell's ideal fluid, the curl of the velocity also equals zero. In electrostatics, the divergence of the electric displacement depends

[1] Letter to R. B. Litchfield, 1858, Campbell and Garnett, *Life,* p. 219.

[2] Sir Joseph Larmor makes the point here developed in his *Aether and Matter,* Cambridge, 1900, p. 75.

upon the volume distribution of the positive and negative charges; in the motion of the fluid, the divergence of the velocity depends upon the distribution of the sources and sinks.

Maxwell himself made no systematic use of vectorial methods. However, in *Electricity and Magnetism,* published in 1873, he stated some of his results in a vectorial notation. And in a paper 'On the Mathematical Classification of Physical Quantities', published in 1871, he suggested some names, including 'curl', for the various operations with Hamilton's differential operator. Maxwell's names were designed to associate certain physical ideas with the mathematical operations.[1] The paper and also *Electricity and Magnetism* foresaw the extensive use of vectorial methods in the analysis of three-dimensional problems in physics. Maxwell regarded vector analysis as the natural language of electromagnetism and of such comparable disciplines as hydrodynamics and elasticity.

> The invention of the calculus of Quaternions is a step towards the knowledge of quantities related to space which can only be compared for its importance, with the invention of triple co-ordinates by Descartes. The ideas of this calculus as distinguished from its operations and symbols, are fitted to be of the greatest use in all parts of science.[2]

The paper on classification suggested a distinction between the physical classification of quantities and the mathematical classification. Maxwell believed that this distinction when fully developed would serve in the correct expression, even detection, of physical analogies.

> . . . if we had a true mathematical classification of quantities, we should be able at once to detect the analogy between any system of quantities presented to us and other systems of quantities in known sciences, so that we should lose no time in availing ourselves of the

[1] 'On the Mathematical Classification of Physical Quantities,' *Proceedings of the London Mathematical Society*, 3, No. 34; reprinted in *Papers*, 2, 263-266

[2] Ibid., *Papers*, 2, 259. See also the introductory section to *Electricity and Magnetism*, § 10. Maxwell writes: 'As the methods of Des Cartes are still the most familiar to students of science, and as they are really the most useful for purposes of calculation, we shall express all our results in the Cartesian form. I am convinced, however, that the introduction of the ideas, as distinguished from the operations and methods of Quaternions, will be of great use in the study of all parts of our subject, and especially electrodynamics, where we have to deal with a number of physical quantities, the relations of which to each other can be expressed far more simply by a few expressions of Hamilton's, than by ordinary equations!'

mathematical labours of those who have already solved problems essentially the same.[1]

The physical classification of a quantity was to be according to the science in which the quantity occurred, while the mathematical classification was to be according to the form of the quantity. Thus temperature may be classified with pressure, for both occur in the science of thermodynamics, but temperature may also be classified with mass, for both are scalar in form. The paper pointed out such other forms of quantities as vectors referred to lines or line integrals, vectors referred to areas or surface integrals, and the different kinds of vector fields.[2]

2

In the above discussion of the nature or character of physical analogy, some mention was made of the uses of this method. Analogy, Maxwell stated in his address to the British Association, was '. . not only a legitimate product of science, but capable of generating science in its turn.' [3] In the course of his writings, Maxwell distinguished, in effect, two ways in which analogy generated science. One, the detection of an analogy made it possible to transfer the solution to a mathematical problem in one branch of physics to another branch of physics. Two, the attempt to make an analogy more complete served to direct entirely new theoretical and experimental inquiries. Maxwell also discussed the doubts sometimes felt as to whether analogy was a condition of scientific inquiry or just an expository device.

The use of physical analogy to transfer the solution to a problem from one branch of physics to another is illustrated by many of the advances in electricity and magnetism. The paper on classification stated, for example, that, for the Italian physicist Mossotti to obtain the mathematical theory of electrostatic induction, he had only to take Poisson's work on magnetic induction, and translate the magnetic language into electric and the French into Italian.[4] Maxwell's review of Kelvin's *Reprint of Papers on Electrostatics and Magnetism* remarked that Kelvin's analogy between electrostatics and heat conduction made

[1] 'On the Mathematical Classification of Physical Quantities,' *Proceedings of the London Mathematical Society*, 3, No. 34, reprinted in *Papers*, 2, 258

[2] Ibid., *Papers*, 2, 257-262. See also *Electricity and Magnetism* §§ 11-19

[3] 'Address to the Mathematical and Physical Sections of the British Association', *Papers*, 2, 227

[4] 'On the Mathematical Classification of Physical Quantities,' *Papers*, 2, 258

the mathematical methods of Fourier available to electrostatics.[1] The analogy between electrostatics and fluid motion transferred problems concerning distributions of sources and sinks into problems concerning distributions of charged bodies. In his third great memoir on electricity, ' A Dynamical Theory of the Electromagnetic Field, ' published in 1864, Maxwell demonstrated that the equations of motion developed for the study of material systems by Lagrange, some hundred years earlier, in the *Mécanique analytique* were also applicable to electric circuits.[2]

The second use of physical analogy is more profound. Analogies are incomplete, and the attempt to make them more complete suggests new directions for inquiry. Further, physical analogies sometimes develop into acceptable physical hypotheses. Maxwell's creation of the mathematical theory of the field and the electromagnetic theory of light nicely illustrates how the method of analogy is, to use Maxwell's own term, ' science-forming '.[3] The memoir ' On Physical Lines of Force ' consciously employed two analogies. One analogy is between a stationary field and a solid under stress. Since the simplest medium which can support transverse wave-motion is an elastic solid, this analogy proved singularly fruitful. The other analogy is between electricity and fluid motion, with its suggestion that Ampère's law be modified to satisfy the equation of continuity. Maxwell's work in kinetic theory illustrates the development of a physical analogy into a physical hypothesis. Kinetic theory began in the root analogy between the observed properties of gases, and a system of small, hard, perfectly elastic spheres, acting upon one another only during impact.

Philosophers of science have made the claim that analogy, in the nineteenth century, did not so much direct inquiry as aid the exposition of discoveries obtained by other means. The comparisons to mechanics made the introduction of the new ideas easier by relating the unfamiliar to the known. But Maxwell, for his part, found it just as hard to believe that Ampère, who presented *his* results directly in mathematical terms, actually obtained these results in such fashion. *Electricity and Magnetism* charged Ampère with erecting a ' scaffolding ' only to remove it prior to publication.

[1] Review of *Reprint of Papers on Electrostatics and Magnetism*, in *Nature*, 7 ; reprinted in *Papers*, 2, 302

[2] ' A Dynamical Theory of the Electromagnetic Field,' *Royal Society Transactions*, 1864 ; reprinted in *Papers*, 1, 526-597

[3] Review of *Reprint of Papers on Electrostatics and Magnetism*, *Papers*, 2, 301

> The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the 'Newton of electricity'. It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics.
>
> The methods of Ampère, however, though cast into an inductive form, does not allow us to trace the formation of the ideas which guided it. We can scarcely believe that Ampère really discovered the law of action by means of the experiments which he describes. We are led to suspect, what, indeed, he tells us himself, that he discovered the law by some process which he has not shown us, and that when he afterwards built up a perfect demonstration he removed all traces of the scaffolding by which he had raised it.[1]

To be sure, Maxwell admitted that analogy also functioned as an aid to the understanding both for the original investigator and for the reader. At the time of publication of the first memoir fluid motion was more familiar to scientific minds than electricity. The *Elementary Treatise on Electricity* suggested that if the history of science had been different, then the various analogies exhibited by electrical phenomena '. . . would have served rather to obscure than enlighten minds'.[2] Further, Maxwell recognised the existence of different kinds of minds engaged in the scientific enterprise. In the address to the British Association he found that the laws of physics may be expressed either symbolically, as relations among abstract quantities, or as physical analogies.

> There are men who, when any relation or law, however complex, is put before them in a symbolical form, can grasp its full meaning as a relation among abstract quantities. Such men sometimes treat with indifference the further statement that quantities actually exist in nature which fulfil this relation. The mental image of the concrete reality seems rather to disturb than to assist their contemplations. . . .
>
> Others, again, are not content unless they can project their whole physical energies into the scene which they conjure up. They learn at what a rate the planets rush through space, and they experience a delightful feeling of exhilaration. They calculate the forces with which the heavenly bodies pull at one another, and they feel their own muscles straining with the effort.

[1] *Electricity and Magnetism*, § 528. See also Maxwell's letter to Kelvin, Nov. 13, 1854, *Origins of Clerk Maxwell's Electric Ideas*, p. 8. The confession of Ampère to which Maxwell refers in the quotation above is in *Théorie des phénomènes électrodynamiques*, p. 9.

[2] *Elementary Treatise*, § 68

> To such men momentum, energy, mass are not mere abstract expressions of the results of scientific inquiry. They are words of power, which stir their souls like memories of childhood.
>
> For the sake of persons of these different types, scientific truth should be presented in different forms, and should be regarded as equally scientific, whether it appears in the robust form and the vivid colouring of a physical illustration, or in the tenuity and paleness of a symbolical expression.[1]

In fine, analogy is a condition of scientific speculation. It was *the* condition for Maxwell, though he was ready to allow other conditions as well.

3

Some of the now classic writers on the philosophy of science touched on the method of physical analogy. For the most part, however, they only raised again the points made by Maxwell himself. Pierre Duhem, in *La Théorie physique*, defined physical analogy as a similarity in mathematical form between two sciences which describe different sets of phenomena.[2] In *Les Théories electriques de J. Clerk Maxwell* he even cited 'On Faraday's Lines of Force' as a prime example of the correct and successful use of physical analogy.[3] Further, *La Théorie physique* distinguished, in effect, between physical analogy and physical hypothesis,[4] and discovered two types of minds engaged in science. These two types, for Duhem, followed national boundaries, with the broad but weak mind of the English reflected in a reliance upon illustration, and the narrow but strong mind of the continental scientist reflected in a taste for purely symbolical expression.[5] Ernst Mach, in *Erkenntnis und Irrtum*, also mentioned the analogy of the first memoir. He remarked upon the advantages of physical analogy, noted that analogies are incomplete, and illustrated some of these considerations using the analogy between light and small disturbances in an elastic solid.[6] Norman R. Campbell, in *Physics: The Elements*, discussed physical analogy and physical hypothesis which he called physical theory. For the physical theory of a given set of phenomena to be adequate, it must be more than a mathematical description. It must display an analogy to some other set of

[1] 'Address to the Mathematical and Physical Sections of the British Association', *Papers*, 2, 219-220

[2] Duhem, *La Théorie physique*, Paris, 1906, Part I, Chapter IV

[3] Pierre Duhem, *Les Théories electriques de J. Clerk Maxwell*, Paris, 1902, pp. 48-58

[4] Duhem, *La Théorie physique*, Part I, Chapter IV [5] Ibid., p. 169

[6] Ernst Mach, *Erkenntnis und Irrtum*, Leipzig, 1905, pp. 217-228, 241

phenomena.[1] Campbell cited as examples of such physical theories the kinetic theory of gases and, in *What is Science?*[2] the electromagnetic theory of light.

To be sure, there are important differences between Duhem and Mach, on the one hand, and Campbell, on the other. Duhem and Mach, in the tradition of Rankine and Ostwald, rejected physical hypotheses as a legitimate goal of physics. They favoured phenomenological theories. In contemporary terms, they wanted to restrict physics to macroscopic theories, eliminating microscopic theories. Upon occasion Maxwell himself was also prepared to discard physical hypotheses. For Maxwell, however, such rejection was not a matter of principle but of evidence. In the memoir 'On Physical Lines of Force', he offered a physical hypothesis for electromagnetic phenomena which was parallel in many respects to the kinetic theory of gases. Maxwell subsequently discarded this hypothesis but retained the kinetic theory.

By way of conclusion, it is well to distinguish between the method of physical analogy and the familiar argument by analogy. The argument by analogy as applied to science looks something like this. Some set of phenomena A is known to be like a set of phenomena B with respect to properties x, y, and z. And set of phenomena A, in addition, is known to possess property w. Therefore, so the argument runs, B must also possess w. The argument by analogy is not deductive; the conclusion does not necessarily follow from the premisses. There is a certain probability, however, that B also possesses w. The measurement of this probability is investigated in inductive logic.

The method of physical analogy differs in an important way from the argument by analogy. Again the set of phenomena A is like the set of phenomena B with respect to properties x, y, and z. And again A, in addition, is known to possess w. Now, does B also possess w? The question comes from the analogy, but the answer must come from further theoretical and experimental investigation. It is not a matter of arguing from old premisses, but of establishing new ones. Success consists either in finding evidence that B does possess w or that B does not possess w. In the first case, the analogy is made more complete; in the second case, the exact limits of the analogy are discovered. Both procedures are 'science-forming', both are parts of the method of physical analogy.

[1] Norman R. Campbell, *Physics: The Elements*, Cambridge, 1920, pp. 120-140

[2] Norman R. Campbell, *What is Science?* London, 1921, Chapter V